

Job Design and Ability as Determinants of Employee Motivation: Developing the Mathematics of Human Motivation via the Law of Escalating Marginal Sacrifice

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Relatively little in the literature explains how job design and employee ability combine to affect employee motivation and satisfaction, and to impact, indirectly, employee performance on the job. This is a theoretical development paper illustrating how job design and ability upgrades of modest magnitude can result in motivation, satisfaction, and performance increases of substantial magnitude. The key to generating major motivation, satisfaction, and performance increases from relatively minor ability/job design upgrades lies in understanding the degree to which job design/ability changes affect the rates of change of performance functions and employee cost functions.

INTRODUCTION

Many now accept that employee-job performance is a function of the design of the job (work) as well as the execution of the person doing the job. Traditionally, the job design component has not been central to the performance model; only recently has it been incorporated as a key performance-determining variable. Job design covers nearly all significant factors that affect performance beyond the employee him/herself. Such dimensions as task structure, authority built into the job, work layout, procedural simplicity, equipment used, degree of task integration, performance feedback mechanisms, resource input, and so on are all elements of job design. We can write the model (Grant, 1999):

$$(1) \text{ Employee-Job Performance} = f(\text{Job Design}; \text{Employee Execution})$$

Employee execution is in turn a function of abilities and motivation, or:

$$(2) \text{ Employee Execution} = g(\text{Abilities}; \text{Motivation})$$

Abilities encompass skills, knowledge and personality traits. Motivation is the amount of effort one exerts to accomplish the job.

THE DIRECT AFFECT OF JOB DESIGN AND ABILITY IMPROVEMENT ON PERFORMANCE

Both job design and ability change can impact performance *directly*. If you improve the design of work, with abilities and motivation remaining constant, performance increases. Similarly, improving the

ability of the employee through training, with job design and motivation remaining untouched, will result in performance increase.

See Figure 1. Improvement in job design/ability will cause a rotational, counterclockwise shift in the performance curve, such as from performance function P_1 to P_2 . With the employee operating at x (the motivational level at which one will operate since there satisfaction is optimized) the *direct* influence of the job design/ability improvement is to boost performance from a_1 , on performance curve P_1 , to a_2 , on performance curve P_2 (Grant, 2004). A job design/ability improvement shifts performance “up” for a given motivation level by increasing the slope of the performance function.

We are assuming, in Figure 1, that the performance function is linear for ease of exposition. In general, performance, as a function of motivation, tends to rise at a decreasing rate. Indeed, at extremely high levels of motivation, performance may begin to decline, in absolute terms, as excessive motivation becomes dysfunctional.

THE INDIRECT AFFECT OF JOB DESIGN AND ABILITY IMPROVEMENT ON PERFORMANCE VIA REWARD FUNCTION CHANGE

Job design and ability can also affect performance *indirectly* through their influence on motivation. In fact, the most significant determinants of motivation are often job design and ability. This is seldom recognized by classical motivation theory, but frequently the most expeditious motivational strategy involves a change in job design and/or ability rather than working with the employee and the perceived rewards and costs that affect motivation but which are exclusive of job design and ability (Helm, 2007).

Consider Figure 2, which illustrates only the reward and cost curves for an employee (these are the same reward and cost curves shown in Figure 1). Though reward functions, as is the case with performance functions, tend to rise at a decreasing rate, the reward curve shown in Figure 2 is linear to avoid complicating discussion. This assumption of linearity takes nothing away from the insights and conclusions forthcoming, however.

The reward and cost equations, represented graphically in Figure 2, are respectively:

$$(3) r = 2.4x$$

Where:

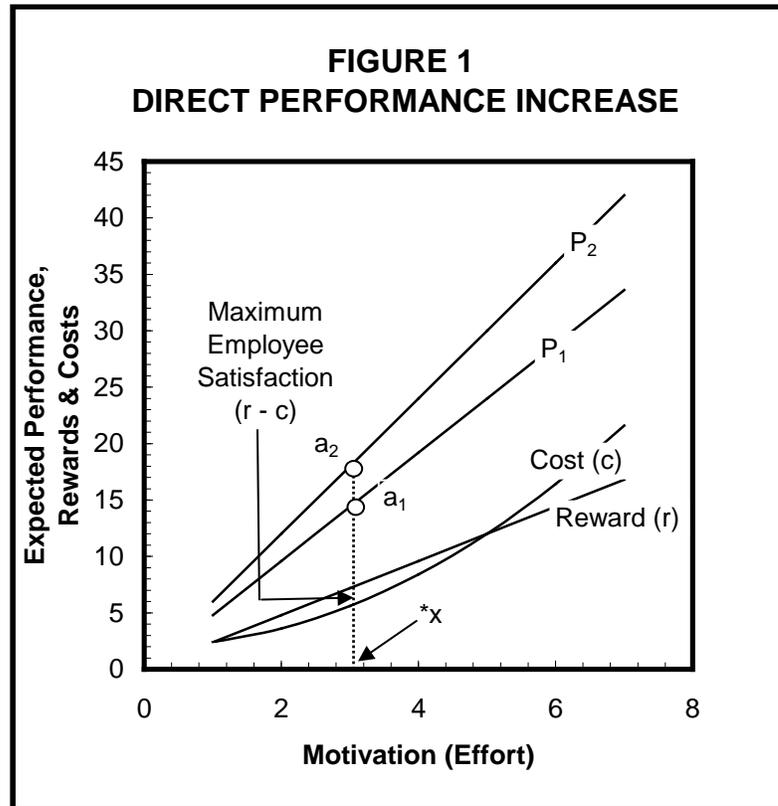
r is perceived expected reward, and
 x is the amount of motivation
(effort) exerted.

And:

$$(4) c = 2 + .4x^2$$

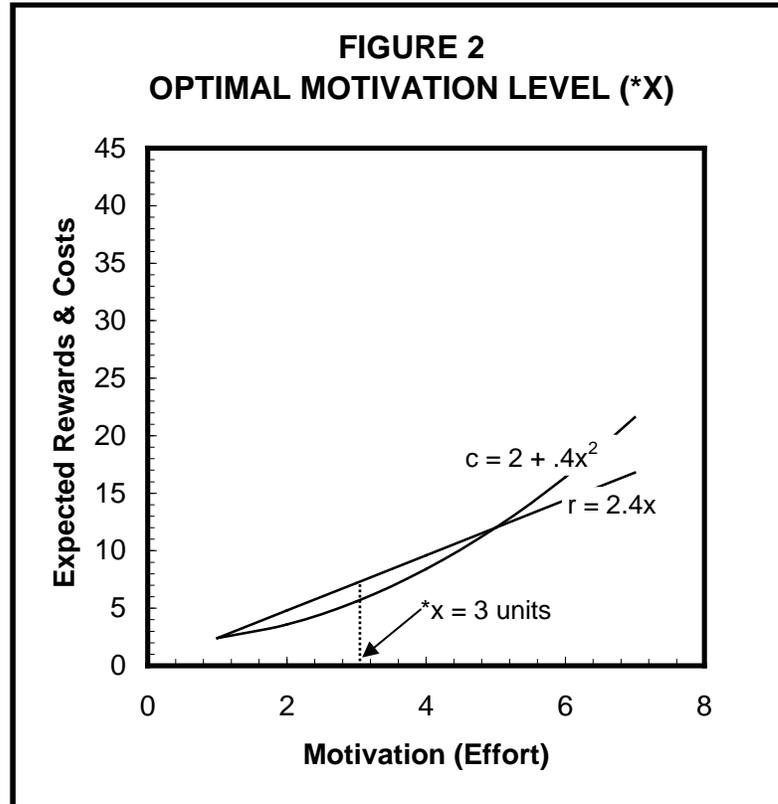
Where:

c is perceived expected cost (expected sum of fatigue, fear, boredom, opportunity costs, stress and so on which rise according to the Law of Escalating Marginal Sacrifice), and
 x is again the amount of effort exerted.



With a job design and/or an ability increase, the performance function increases in slope, from P_1 to P_2 . This results in a direct increase in performance, from a_1 to a_2 . It is assumed here that both the performance and reward functions are linear. It is also assumed that the reward function is independent of the performance function, therefore, the reward function is unaffected by the performance function shift.

The employee operates at effort level $*x$, under both performance functions, because the difference between rewards and costs--that is, satisfaction--is maximized there.



The reward function here is shown as linear, as in the other graphs, but typical reward functions will exhibit a declining rate of change as higher motivation levels are approached. This is due to the Law of Declining Marginal Utility and to the Law of Diminishing Returns. However, whether the reward function exhibits a constant slope or a decreasing slope, the concepts at the heart of this analysis are unaffected.

The cost function here exhibits the typical increasing slope, with increasing motivation, as explained by the Law of Escalating Marginal Sacrifice. The employee operates at 3 units (*x) of effort where marginal reward and marginal cost equate and satisfaction is maximized.

The optimal motivation for the individual occurs at the effort level where the rates of change of the reward function and cost function are equal, or where expected satisfaction is perceived to be maximized. For optimization:

$$(5) \partial r/\partial x = \partial c/\partial x$$

Or, from (3) and (4):

$$(6) 2.4 = .8x$$

Solving for x gives the optimal x, *x:

$$(7) *x = 3 \text{ units of effort}$$

Let us now add a performance function (where performance is, say, measured as expected output value) which is two times the reward function in (3)—or, the expected reward value is one half of the expected output value (Woolnough, 2005). That is:

$$(8) P = 2(2.4)x \text{ or, } r = 4.8x/2$$

Where:

P is performance,
x is motivation, and
r is rewards.

In other words, rewards are *dependent* on performance. Such dependency is common place. Generally, some portion of every employee's intrinsic and extrinsic rewards is a function of performance. The performance, reward, and cost functions given above in (8), (3) and (4) respectively are shown together in Figure 3.

Now, suppose there is an upgrade in employee abilities and/or in job design. The direct affect of this is to angularly shift (as opposed to a lateral shift) the position of the performance function such that the slope of that function is greater. See Figure 4, where it is assumed the performance function “jumps” to a slope of six—from P_1 to P_2 , or from $P_1 = 4.8x$ to $P_2 = 6x$. So at the effort level, *x (3 units) exerted in Figure 3, performance improves from a_1 (14.4) to a_2 (18). This is the *direct* affect of design/ability improvement on performance initially illustrated in Figure 1.

But what happens to employee motivation? See Figure 5. With the shift in the performance function (P_1 to P_2), the reward function shifts (since rewards are dependent on performance) to a new position with a greater slope—that is from r_1 to r_2 . When the rate of change of the *reward* function increases, motivation will increase. The new motivation is found by:

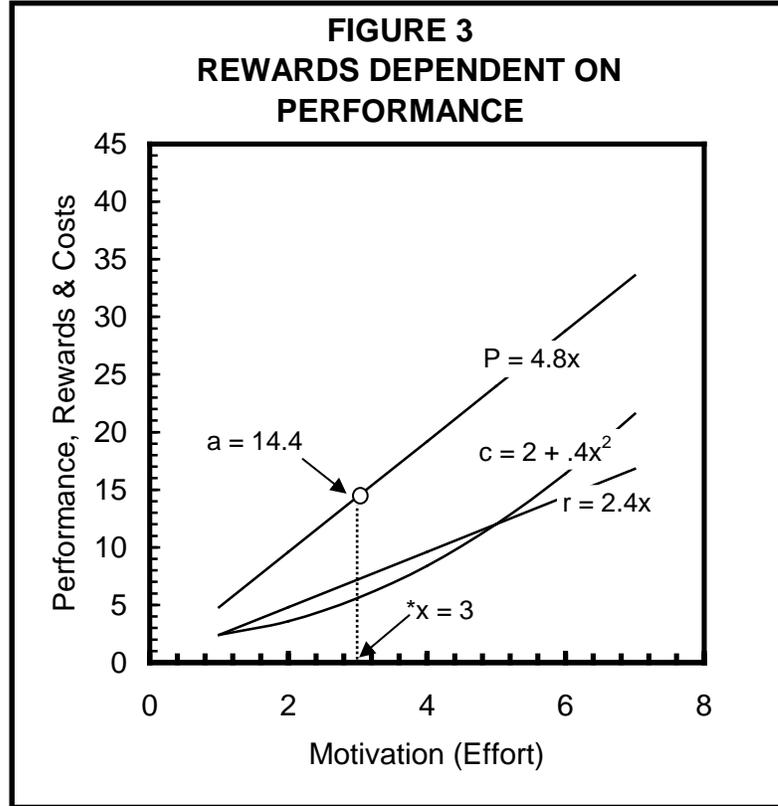
$$(9) \partial r_2/\partial x = \partial c/\partial x$$

Where:

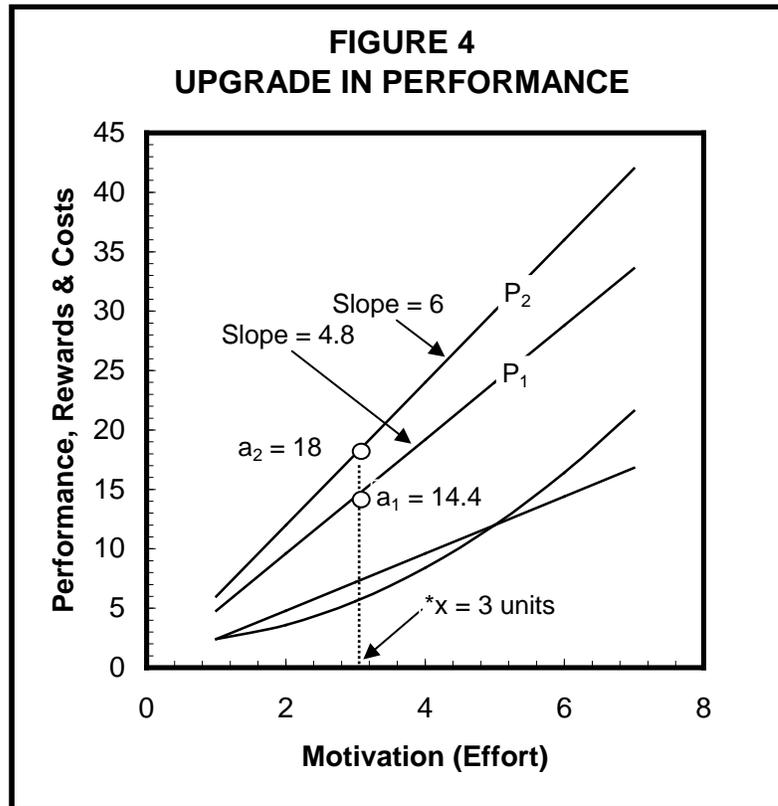
r_2 is the new reward function,
c is employee cost, and
x is motivation.

Or, since $r_2 = P_2/2 = 6x/2 = 3x$, we have:

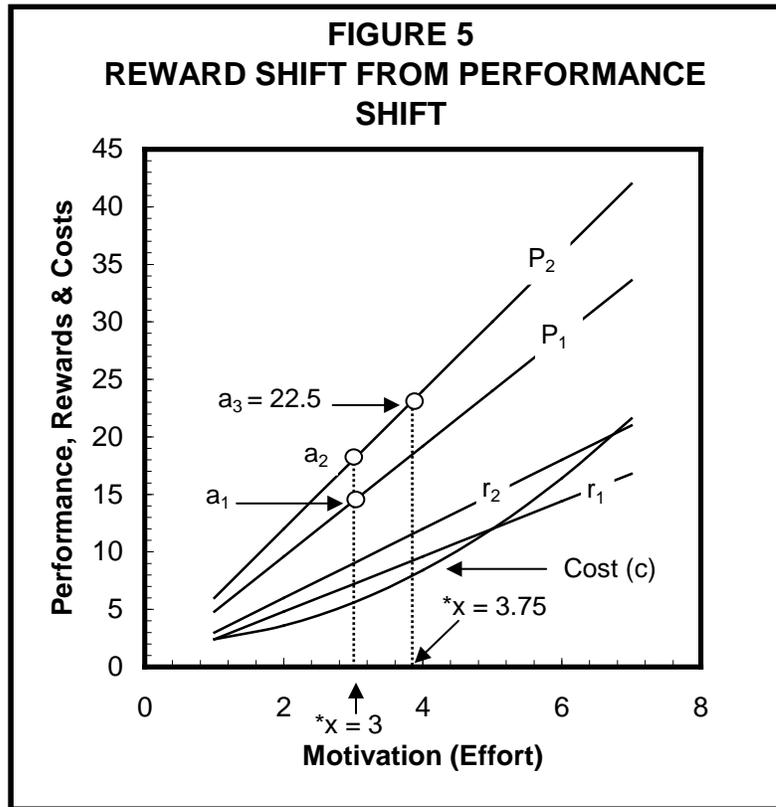
$$(10) 3 = .8x$$



The employee operates at *x, where there is the greatest difference between the reward and cost functions. At *x the production (performance) level is 14.4 units (a). The reward function is one half the performance function here--a simple, clear dependency relationship.



The functions in this figure are the same as those in Figure 1 but with the "numbers" attached. Here the performance function rate of change is illustrated to shift from 4.8 to 6 due to a job design/ability improvement. Given the specific functions, this yields a direct performance gain from a_1 to a_2 (from 14.4 units to 18 units).



Here the reward function is shown as having shifted (from r_1 to r_2) as a result of the performance curve shift (P_1 to P_2) since rewards are dependent on performance. The reward curve shift causes motivation to shift to a higher level (from $*x = 3$ to $*x = 3.75$) because the slopes of the reward and cost functions equate at a greater magnitude of effort--that is, satisfaction is maximized at a higher level of effort. The higher motivation amplifies performance, moving it from a_2 to a_3 .

Solving for *x:

$$(11) \ *x = 3.75 \text{ units of effort}$$

Motivation increases from 3 units, in equation (7), to 3.75 units due to an angular translation of the reward function generated by higher performance caused by a job design/ability improvement.

But if motivation goes to 3.75 units, the value of output increases to a_3 (22.5) in Figure 5. The original production increase from job design/ability improvement is “amplified” from a_2 (18) to a_3 (22.5) by an increase in motivation, from *x (3 units) to *x (3.75 units) accompanying the job design/ability upgrade. This increase in performance from 18 to 22.5 (*along* the P-function) units is due to the *indirect* affect of job design/ability upgrade on performance—i.e. the job design/ability improvement affects motivation which in turn affects performance.

REWARD FUNCTION INDEPENDENCE FROM PERFORMANCE FUNCTION

We have considered reward functions to be dependent on performance (they typically are, to a degree) and to shift as a result of performance function shift. However, it should be noted that job design/ability changes may be “felt” only by the reward function, leaving the performance function “in place”. In such cases, with a job design/ability improvement, you experience an increase in reward function slope (and perhaps reward function “height” as well) which causes higher motivation and an increase in performance *along* the performance function but no *direct* increase in performance due to an angular displacement.

THE INDIRECT AFFECT OF JOB DESIGN AND ABILITY IMPROVEMENT ON PERFORMANCE VIA COST FUNCTION CHANGE

Job design and ability upgrades, however, do not affect only the performance and reward functions. Improvements in job design/ability can also impact one’s perceived cost function by “bending it down”. When job designs get better, fatigue, boredom, stress and so on can easily decrease. Similar things happen when employee abilities are improved. The typical cost function will escalate at a slower rate when the job becomes easier, safer, clearer, more fun, better understood, and so on.

Suppose a job design/ability improvement shifts the perceived cost function (c) in Figure 5 to:

$$(12) \ c_2 = 2 + .2x^2$$

This new cost function has been added to the set of functions from Figure 5 and appears in the function mix in Figure 6. With the new “lower slope” cost function, motivation moves still higher to where:

$$(13) \ \partial r_2 / \partial x = \partial c_2 / \partial x$$

Or, from (10) and (12):

$$(14) \ 3 = .4x$$

Solving for *x:

$$(15) \ *x = 7.5$$

Motivation takes a relatively large “jump” from 3.75 units (when only the reward function slope increase was recognized as a result of the job design/ability upgrade) to 7.5 units of effort when the decrease in cost function rate of escalation is recognized and coupled with the reward function slope

increase. This additional motivation creates a further “amplification” in production from a_3 (22.5) to a_4 (45).

By boosting the quality of job design and ability, performance not only improves directly, up to a_2 from a_1 , but also improves from a_2 to a_4 as a result of the higher motivation generated from reward and cost function realignment. This is the total *indirect* affect of job design/ability improvement on performance.

RESPONSIVENESS OF MOTIVATION, SATISFACTION, AND PERFORMANCE TO JOB DESIGN AND ABILITY CHANGE

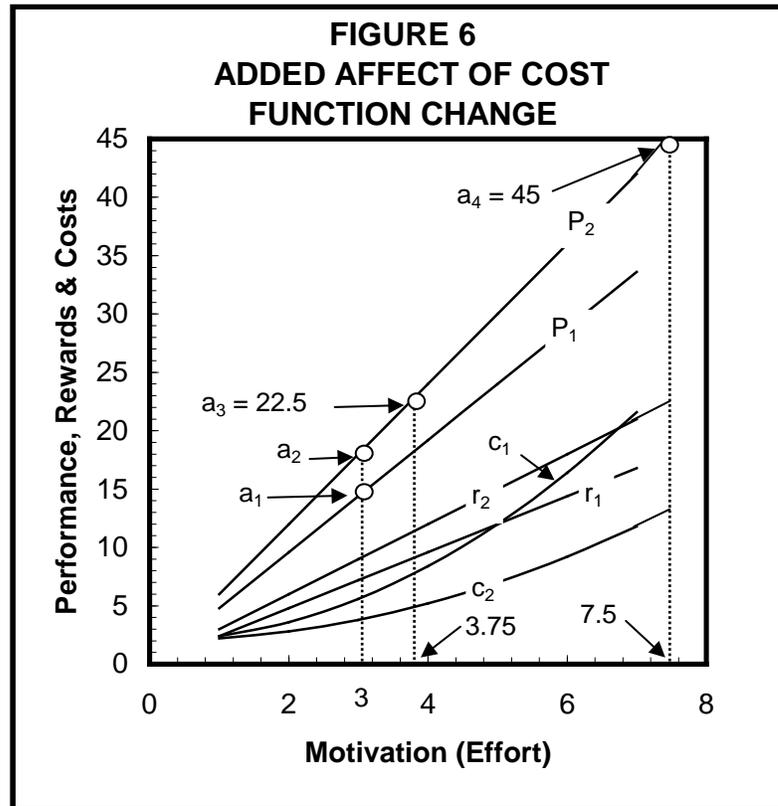
One powerful insight here is that relatively small changes in the slopes of performance and sacrifice functions, caused by job design/ability changes, can have a significant impact on motivation, satisfaction and performance. Indeed, depending on the exact nature of the functions, the indirect improvement in performance—an improvement along the performance function—can be greater than the direct improvement in performance resulting from an angular performance curve shift.

To assess the sensitivity of motivation to job design/ability upgrade, we can do the following leverage calculation (Salvatore, 1996):

$$(16) L_x = \% \Delta^*x / [(\% \Delta \partial P / \partial x + \% \Delta \partial c / \partial x) / 2]$$

Where:

L_x is the number of times greater the percent change in the optimal motivation is than the average of the percent changes in the rates of change of the performance and cost functions,
 $\% \Delta^*x$ is the percent change in motivation,
 $\% \Delta \partial P / \partial x$ is the percent change in the slope of the performance function, and
 $\% \Delta \partial c / \partial x$ is the percent change in the slope of the cost function.



Lowering the rate of escalation of the cost function causes marginal rewards (slope of r_2) and marginal costs (slope of c_2) to equate at a still higher level of effort ($*x = 7.5$). Motivation is now so high that performance "shoots up" along the performance function (P_2) from a_3 to a_4 (from 22.5 units to 45 units).

Notice, motivation increases generated by job design/ability upgrade cause performance to increase along performance functions (the indirect affect of job design/ability improvement on performance), while the direct affect of job design/ability improvement on performance is to cause the performance function to shift, counterclockwise, to a new position.

For the above illustration:

$$(17) \% \Delta^* x = (7.5 - 3)/3 = 1.5 \text{ or } 150\%$$

$$(18) \% \Delta \partial P / \partial x = (6 - 4.8)/4.8 = .25 \text{ or } 25\%$$

$$(19) \% \Delta \partial C / \partial x = (.8x - .4x)/.8x = .5 = 50\%$$

Therefore:

$$(20) L_x = 1.5 / [(.25 + .5)/2] = 1.5 / .375 = 4$$

This means a percent change in the performance and cost function slopes due to job design/ability upgrade will generate four times that percent change in motivation. In other words, motivation is rather highly sensitive, in the illustration, to a job design/ability change.

To determine how sensitive satisfaction is to a given job design/ability change, we can calculate satisfaction leverage:

$$(21) L_s = \% \Delta^* S / [(\% \Delta \partial P / \partial x + \% \Delta \partial C / \partial x) / 2]$$

Where:

L_s is the number of times greater the percent change in the optimal satisfaction is than the average of the percent changes in the rates of change of the performance and cost functions, and $\% \Delta \partial P / \partial x$ and $\% \Delta \partial C / \partial x$ are as before.

For the above illustration:

$$(22) \% \Delta^* S = (6.44 - 1.6) / 1.6 = 3.025 \text{ or } 302.5\%$$

Therefore:

$$(23) L_s = 3.025 / .375 = 8.07$$

This means the change in percent satisfaction in the above illustration is about eight times the average of the angular percent changes in the performance and cost functions. Or, satisfaction here is extremely responsive to the job design/ability changes that generated the angular performance and cost function changes.

Similarly, the responsiveness of performance to job design/ability improvement can be figured as follows:

$$(24) L_p = \% \Delta P / [(\% \Delta \partial P / \partial x + \% \Delta \partial C / \partial x) / 2]$$

Where:

L_p is the number of times greater the percent change in the performance is than the average of the percent changes in the rates of change of the performance and cost functions, and $\% \Delta \partial P / \partial x$ and $\% \Delta \partial C / \partial x$ are as before.

For the above illustration:

$$(25) \% \Delta P = (45 - 14.4) / 14.4 = 2.125 \text{ or } 212.5\%$$

Therefore:

$$(26) L_p = 2.125 / .375 = 5.67$$

This reveals that the percent change in performance compared to the percent changes in the rates of change of the performance and cost functions, due to the job/design ability improvement, is very high. And a closer look shows the indirect magnitude of change in performance is much larger than the direct change.

CONCLUSIONS

What we have shown here is that “bending up” one’s perceived, expected reward function and “bending down” one’s perceived, expected cost function through relatively modest job design and ability improvements can significantly increase employee motivation and satisfaction, and significantly amplify any performance improvements that occur directly from job design/ability improvements. The exact magnitude of improvements in performance, satisfaction, and motivation will depend on the magnitude of changes in the slopes of the performance, reward, and cost curves resulting from changes in ability and job design.

The greater the direct change in performance from an ability/job design change, and the greater the degree to which rewards are dependent on performance, the greater any indirect changes in performance due to motivation improvement. Further, very slight changes in cost functions resulting from ability/job design change can have a large impact on motivation, satisfaction, and performance.

The leverages calculated for the example presented simply show that it is quite possible to have relatively large changes in performance, satisfaction, and motivation associated with relatively small changes in performance and cost function rates of change (slopes) which are stimulated by ability/job design upgrades. Bottom line: Job design and ability are major players in determining employee motivation and satisfaction, as well as in determining performance. Job design and ability both affect the rates of rise of reward and cost functions. Upgrading job design/ability causes motivation to shift to a higher level because maximum satisfaction occurs at a higher level when the “spread” between rewards and costs is increased. The general strategy for management to pursue in boosting employee motivation is to “bend up” the reward function and/or to “bend down” the cost function.

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