Comparing Two Exchange Rate Regimes Under Purchasing Price Disparity

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The theoretical and empirical literatures do not give a clear answer about the superiority of flexible exchange rate regime over fixed exchange rate regime when purchasing power parity (PPP) condition fails to hold. The flexible exchange rate regime is generally shown to be superior, assuming PPP. This study is a fresh examination of the popular assertion that Flexible Exchange Rate regime outperforms other Exchange Rate regimes. We analyze the effect of the violation of purchasing power parity combined with deviations in output target and real exchange rate from their long-run equilibrium values on government’s decision domain, which we call government’s loss function. This study uses a government loss function, for the most part derived from Barrow and Gordon (1983), to compare two exchange rate regimes under different PPP conditions. The results of this study confirm that a flexible exchange rate system indeed performs better if PPP holds. However, this implication may not be true when PPP doesn’t hold, even if the cost of exchange rate change is zero. A major finding of this study, in sharp contrast to Obstfeld (1996), is that, if PPP does not hold, the flexible exchange rate system cannot be guaranteed to perform better unless two additional conditions are met: (a) output target is fully adjusted to its long-run equilibrium value; and (b) the long-run real exchange rate is lower than its long-run equilibrium value. If these conditions are not met, the result is ambiguous. Since the above two conditions may not always hold, the implication of this study is quite significant and offers the opportunity for future empirical research.

INTRODUCTION

Since the collapse of Breton Wood system, policy makers have been searching for a stable international monetary system that could promote international trade and encourage long term investment. As a working solution to this issue, managed exchange rate system allowing deviations within a very narrow band have been adopted. One of the outcomes of such an attempt was the initiation of European Monetary System, an arrangement wherein the member countries, including most nations of the European Economic Community (EEC) were allowed to manage their currencies depending on economic fundamentals and shocks within a band around the current value called target zones (Krugman, 1991). The presumed benefit is that exchange rate stability is supposed to bring about price stability, which
enhances trade and consequently economic prosperity. Implicit in this idea is the assumption that the purchasing power parity (henceforth referred to as PPP) always holds. If PPP holds, we have \( P = P^*E \), where, \( P \) is the domestic price, \( P^* \) is the foreign (world) price, and \( E \) is the exchange rate. Stability in \( E \), therefore, translates into stability in \( P \).

Similarly, a fixed exchange rate is also viewed as the measure to equalize interest rates across borders as shown by the interest rate parity equation: \( R = R^* + \frac{E^e - E}{E} \), where \( R \) and \( R^* \) are domestic and foreign rates of interest respectively, \( E \) is the current exchange rate, and \( E^e \) is the expected exchange rate.

As a fixed exchange rate eliminates the differential between the current and expected exchange rates, it equalizes the interest rates across the trading countries. Since economic fundamentals do not change as rapidly as people’s expectations, such an exchange rate system can ensure price stability, smooth flow of international trade and capital.

Obstfeld (1996) argues that the economic agents’ expectation is influenced by government’s resources, rather than its current action or commitment. Government’s possible future action depends on relative size of losses under different policy regimes and, although a fixed exchange rate system can bring about price stability, the flexible exchange rate system is a more attractive alternative as long as the cost of the exchange rate adjustments is not very high. This argument depends on two assumptions: (i) PPP holds, and (ii) a fixed exchange rate system can successfully limit people’s expectation. The PPP condition can, however, fail due to several reasons, such as deviations from the Law of One Price (LOP), the presence of non-traded goods, and the terms of trade effects of home bias in consumption. PPP puzzles, a common term for two anomalies of real exchange rates, indicate long-run PPP failures as well (Mussa 1986; Rogoff, 1996; Taylor, Peel, and Sarno, 2001). Additionally, Hyrina and Serletis (2010) used Lo’s modified R/S statistic and Hurst exponent to show that PPP did not hold under currency exchanges between four countries.

So what happens if these assumptions do not hold? Can a target zone system still sustain or does it need a continuous realignment, which is clearly a failure of the target zone system? It is pertinent to ask, “Is a flexible exchange rate system between two currencies always better even if purchasing power parity does not hold between the two countries?”

The relevant theoretical literature does not fully answer these questions whereas the empirical researches have provided mixed results. Flexible exchange rates were optimal, according to Obstfeld (1996) whether deviations from PPP are due to deviations from the LOP or due to the presence of non-traded goods. In contrast, Devereux and Engel (2003) illustrated that, if PPP fails because of deviations from the LOP arising from sticky prices in local currency, then fixed exchange rates are optimal even in the presence of country-specific shocks. These studies do not establish the superiority of one exchange rate regime over the other when PPP condition does not hold and output target and real exchange rate deviate from their long-run equilibrium values. This paper, therefore, is devoted to analyzing the effect of the violation of PPP along with the deviation of output target and real exchange rate from their long-run equilibrium values on government’s decision domain, which we term government’s loss function.

THE MODEL

The model assumes a typical government loss function following Barrow and Gordon (1983) with some modifications. The loss function is of the following form:

\[
L = (Y - KY^*)^2 + \beta \left( \Pi^2 \right) + c(\varepsilon),
\]

where, \( Y \) is the output level, \( Y^* \) is the targeted output level, \( \Pi \) is the rate of inflation, \( c(\varepsilon) \) is the cost of changing the exchange rate, \( \varepsilon \) is the exchange rate, and \( K \) and \( \beta \) are assigned weights. The first squared term in the loss function is the quadratic approximation of the welfare loss of being away from targeted output level. Therefore, the output deviation enters the government loss function because it causes
unnecessary economizing on real balance, which generates costs of price change and even increases endogenous relative price uncertainty (Benabou, 1988). The second term in the equation is the rate of inflation. An unanticipated inflation is costly and socially undesirable because it increases relative price variability (CuKiermann, 1984). The third term is the cost of changing exchange rate. Excessive short-run fluctuations in exchange rates under a flexible exchange rate system may be costly in terms of higher frictional unemployment if they lead to over-frequent attempts at reallocating domestic resources among the various sectors of the economy.

The output function is represented by the augmented Phillips curve as follows:

\[ Y_t = \bar{Y} + \alpha (\Pi_t - \Pi^e_t) + u_t, \]  

\[ (2) \]

where, \( Y_t \) is the output level, \( \bar{Y} \) is the long-run output level, \( \Pi_t \) and \( \Pi^e_t \) are actual and expected inflation rates respectively, and \( u_t \) is the output shock. Other assumptions of this model are as the following:

Purchasing power parity condition: \( e_t - p_t + p^*_t = q_t \)  

\[ (3) \]

Movement of real parity condition: \( q_t - q^e_{t-1} = \lambda (\zeta - q_{t-1}) + v_t \)  

\[ (4) \]

Aggregate demand function: \( m_t - p_t = \beta h_t - \gamma + \mu_t \)  

\[ (5) \]

Uncovered interest parity condition: \( i_t = i^*_t + \epsilon_t \)  

\[ (6) \]

where, \( v_t \sim N(0, \sigma_v^2) \) \[ (7) \]

\( u_t \sim N(0, \sigma_u^2) \) \[ (8) \]

\( \mu_t \sim N(0, \sigma_\mu^2) \) \[ (9) \]

\( \epsilon_t = e_t - e_{t-1} \sim N(0, \sigma_\epsilon^2) \) \[ (10) \]

\( K, \beta, \alpha, \lambda, h, \gamma > 0 \) \[ (11) \]

The variables \( p_t \) and \( p^*_t \) are domestic and foreign price levels respectively; \( q_t \) is the real exchange rate; \( m_t \) is the nominal money supply; \( i_t \) and \( i^*_t \) are domestic and foreign interest rates respectively; and \( u_t, v_t, \) and \( \mu_t \) are output, real exchange rate and demand shocks respectively. Similarly, \( \zeta \) is long-run equilibrium exchange rate. Based on the above assumptions, we derive respective loss functions under flexible and fixed exchange rate systems. The complete derivation is given in the appendix.

The Loss Functions

\[ L^{flex} = \frac{\beta}{\alpha + \beta} (\bar{Y} - KY^* - \alpha e^e_t + u_t + \alpha \lambda (\zeta - q_{t-1}))^2 \]  

\[ (7) \]

\[ L^{fix} = \{ \bar{Y} - KY^* - \alpha e^e_t + u_t - \alpha v_t \}^2 + \beta \lambda^2 (\zeta - q_{t-1})^2 \]  

\[ (8) \]

Taking unconditional expectation yields,

\[ E(L^{flex}) = \frac{\beta}{\alpha + \beta} (\bar{Y} - KY^* - \alpha e^e_t)^2 + \frac{\beta \alpha^2}{\alpha + \beta} \lambda^2 (\zeta - q_{t-1})^2 + \frac{\beta}{\alpha + \beta} \sigma_u^2 \]

\[ + \frac{2\beta \alpha}{\alpha + \beta} \lambda (\zeta - q_{t-1})(\bar{Y} - KY^* - \alpha e^e_t) \]  

\[ (9) \]
\[ E(L^{\text{Fix}}) = (\bar{Y} - KY^* - \alpha \varepsilon_{t})^2 + \sigma_u^2 + \alpha^2 \sigma_v^2 + \beta \lambda^2(\zeta + q_{t-1})^2 + \beta \sigma_v^2 \]  

(10)

The term \( c(\varepsilon) = c(\varepsilon_t - \varepsilon_{t-1}) \) is the cost due to the change in exchange rate. This cost enters only into the loss function under flexible exchange rate system because excessive short-run fluctuations may lead to higher frictional unemployment caused by over-frequent reallocation of domestic resources. In this setup, the monetary authority will be tempted to take resort to the flexible exchange rate system when the effect of \( u_t \) (output shock) and/or \( v_t \) (real exchange rate shock) is so high that \( E(L^{\text{Flex}}) + \bar{c}(\varepsilon) > E(L^{\text{Fix}}) \) or so low that \( E(L^{\text{Flex}}) + \bar{c}(\varepsilon) < E(L^{\text{Fix}}) \), where \( \bar{c}(\varepsilon) \) is the highest value and \( \underline{c}(\varepsilon) \) is the lowest value of \( c(\varepsilon) \). Suppose, \( c^*(\varepsilon) \) is such that,

\[ E(L^{\text{Flex}}) + \bar{c}(\varepsilon) = E(L^{\text{Fix}}) \]

(11)

Substituting equation (9) and (10) into (11) yields,

\[
\frac{\beta}{\alpha^2 + \beta} \left( \bar{Y} - KY^* - \alpha \varepsilon_{t} \right)^2 + \frac{\beta \alpha^2}{\alpha^2 + \beta} \lambda^2(\zeta + q_{t-1})^2 + \frac{\beta}{\alpha^2 + \beta} \sigma_u^2 \\
+ 2\frac{\beta \alpha}{\alpha^2 + \beta} \lambda(\zeta - q_{t-1}) \left( \bar{Y} - KY^* - \alpha \varepsilon_{t} \right) + c^*(\varepsilon)
\]

\[
= \left( \bar{Y} - KY^* - \alpha \varepsilon_{t} \right)^2 + \sigma_u^2 + \alpha^2 \sigma_v^2 + \beta \lambda^2(\zeta + q_{t-1})^2 + \beta \sigma_v^2 \\
\Rightarrow - \frac{\alpha^2}{\alpha^2 + \beta} \left( \bar{Y} - KY^* - \alpha \varepsilon_{t} \right)^2 - \frac{\beta^2}{\alpha^2 + \beta} \lambda^2(\zeta - q_{t-1})^2 - \frac{\alpha^2}{\alpha^2 + \beta} \sigma_u^2 \\
+ 2\frac{\beta \alpha}{\alpha^2 + \beta} \lambda(\zeta - q_{t-1}) ( \bar{Y} - KY^* - \alpha \varepsilon_{t} ) + c^*(\varepsilon) = (\alpha^2 + \beta) \sigma_v^2
\]

\[
\Rightarrow \sigma_v^2 = - \frac{\alpha^2}{(\alpha^2 + \beta)^2} \left( \bar{Y} - KY^* - \alpha \varepsilon_{t} \right)^2 - \frac{\beta^2}{(\alpha^2 + \beta)^2} \lambda^2(\zeta - q_{t-1})^2 \\
- \frac{\alpha^2}{(\alpha^2 + \beta)^2} \sigma_u^2 + 2\frac{\beta \alpha}{(\alpha^2 + \beta)^2} \lambda(\zeta - q_{t-1}) ( \bar{Y} - KY^* - \alpha \varepsilon_{t} ) + \frac{c^*(\varepsilon)}{\alpha^2 + \beta}
\]

(12)

Rearranging equation (12) yields,

\[
c^*(\varepsilon) = \frac{\alpha^2}{\alpha^2 + \beta} (\bar{Y} - KY^* - \alpha \varepsilon_{t})^2 + \frac{\beta^2}{\alpha^2 + \beta} \lambda^2(\zeta - q_{t-1})^2 + \frac{\alpha^2}{\alpha^2 + \beta} \sigma_u^2 \\
- \frac{2\beta \alpha}{\alpha^2 + \beta} \lambda(\zeta - q_{t-1}) ( \bar{Y} - KY^* - \alpha \varepsilon_{t} ) + (\alpha^2 + \beta) \sigma_v^2
\]

(13)
Since \( c^*(\epsilon) \) is the critical value which equalizes \( E(L_{\text{Flex}}) \) and \( E(L_{\text{Fix}}) \), \( c^*(\epsilon) > 0 \) implies \( E(L_{\text{Flex}}) < E(L_{\text{Fix}}) \), while \( c^*(\epsilon) < 0 \) implies \( E(L_{\text{Flex}}) > E(L_{\text{Fix}}) \). Dynamic consistency requires that the government change the exchange rate whenever \( c^*(\epsilon) > 0 \). That is, the fixed exchange rate system is sustainable as long as \( c^*(\epsilon) <= 0 \).

From equations (9) and (10), it is clear that the expected loss in both regimes is an increasing function of real exchange rate deviation (i.e. \( \zeta - q_{t-1} \)). The real exchange rate deviation, however, may cause more or less loss in flexible exchange rate system compared to that in fixed exchange rate system. Under PPP, the loss function under both regimes remains unaffected by real exchange rate deviation. So, if the cost of exchange rate change is negligible, the loss under flexible exchange rate system will be less than that under fixed exchange rate system. However, this is no longer valid when PPP does not hold. To demonstrate, we subtract equation (9) from (10), which yields,

\[
E(L_{\text{Fix}}) - E(L_{\text{Flex}}) = \frac{\alpha^2}{\alpha^2 + \beta} (\bar{Y} - KY* - \alpha \epsilon)^2 + \frac{\beta^2}{\alpha^2 + \beta} \lambda^2 (\zeta - q_{t-1})^2 + \frac{\alpha^2}{\alpha^2 + \beta} \sigma^2_u
- \frac{2\beta \alpha}{\alpha^2 + \beta} \lambda (\zeta - q_{t-1}) (\bar{Y} - KY* - \alpha \epsilon^t) + (\alpha^2 + \beta) \sigma^2_v,
\]

assuming the cost of exchange rate change to be negligible (i.e. \( c(\epsilon) = 0 \)). From the observation of equation (14), it is obvious that there is no guarantee that \( E(L_{\text{Fix}}) - E(L_{\text{Flex}}) > 0 \), even if we assume that the cost of exchange rate change is zero unless two additional conditions are met. If output target is fully adjusted to the long run equilibrium output level i.e. \( \bar{Y} = KY* \), and the real exchange rate is lower than its long-run equilibrium value i.e. \( q_{t-1} < \zeta \), then, from equation (14), it is clear that \( E(L_{\text{Fix}}) > E(L_{\text{Flex}}) \). That is the expected loss under a fixed exchange rate system outweighs the expected loss under a flexible exchange rate system if these two conditions are met.

If PPP holds, then \( q_{t-1} = 0 \) and, therefore, \( \zeta = 0 \). Thus, the negative term on the right hand side of equation (14) drops out, and we have \( E(L_{\text{Fix}}) > E(L_{\text{Flex}}) \). The expected loss under a fixed exchange rate system, consequently, is always greater than that under a flexible exchange rate system when PPP holds. These results can be summarized in the form of the following propositions:

**Proposition 1** - Under purchasing power parity, a flexible exchange rate system always performs better.

**Proposition 2** - Under purchasing power disparity, a flexible exchange rate system performs better only if output target is adjusted to its long-run equilibrium value and the real exchange rate is lower than its long-run value. If these conditions do not hold under purchasing power disparity, then the superiority of a flexible exchange rate system cannot be claimed.

**CONCLUSION**

Price stability is optimal for the long-term prosperity of an economy, and has received high importance in recent studies on macro-economic policy. Obstfeld (1996) argued that, no matter what the government’s current action is, the economic agents’ decision or expectation is influenced by government’s resources rather than its current action or commitment. A government’s potential future action depends on relative size of losses under different policy regimes. Obstfeld further maintains that, although a fixed exchange rate system can bring about price stability, the government always has an incentive to go for the flexible exchange rate system, as long as the cost of changing the exchange rate is not high.
We have shown, however, that this assertion is valid only under purchasing power parity condition. Even if the cost of the exchange rate is zero, the implications drawn by Obstfeld may be accurate if PPP doesn’t hold. Under purchasing power disparity, a flexible exchange rate system can be assured to do better, but only if the output target is fully adjusted to its long-run equilibrium value and its long-run real exchange rate is lower than its long-run equilibrium value.

**APPENDIX**

Derivation of the Loss Functions

Lagging equation (3) by one period and subtracting it from the original equation yields,

\[ e_t - e_{t-1} - p_t + p_{t-1} + p_t^* - p_{t-1}^* = q_t - q_{t-1} \]

or

\[ p_t - p_{t-1} = e_t - e_{t-1} + p_t^* - p_{t-1}^* - (q_t - q_{t-1}) \]

where, \( \Pi_t = e_t + p_t^* - p_{t-1}^* - (q_t - q_{t-1}) \),

\[ \Pi_t = e_t - (q_t - q_{t-1}) \]  
(a1)

Substituting equation (4) into above yields,

\[ \Pi_t = e_t - \lambda (\xi - q_{t-1}) - v_t \]  
(a2)

Taking conditional expectation of equation (a2) based on t period yields,

\[ E_t \Pi_t = E_t e_t - E_t \lambda (\xi - q_{t-1}) - E_t v_t \]

\[ = E_t e_t - \lambda (\xi - E_t q_{t-1})) - E_t v_t \] or

\[ \Pi_t^e = e_t^e - \lambda (\xi - q_{t-1}) \]  
(a3)

Because, at the beginning of period t, q_{t-1} is already realized and E_t q_{t-1} = q_{t-1}. Subtracting equation from (a2) yields,

\[ \Pi_t - \Pi_t^e = e_t - \lambda (\xi - q_{t-1}) - v_t - [e_t^e - \lambda (\xi - q_{t-1})] \]

\[ = e_t - e_t^e - v_t \]  
(a4)

Substituting equation (a4) into (2) yields,

\[ Y_t = \bar{Y} + \alpha [e_t - e_t^e - v_t] + u_t \]  
(a5)
Substituting equation (a2) and (a5) into (1) and ignoring c(e) for the time being yields,

\[ L = \{ Y + \alpha(e_t - e_t^e - v_t) + u_t - KY^*\}^2 + \beta[e_t - \lambda(\zeta - q_{t-1})] - v_t \]  

The first order condition for the minimization of equation (a6) requires the following:

\[ \frac{\partial L}{\partial \mathcal{E}_t} = 2\{ Y + \alpha(e_t - e_t^e - v_t) + u_t - KY^*\}a + 2\beta[e_t - \lambda(\zeta - q_{t-1})] - v_t = 0 \]

This implies the followings respectively:

\[ \Rightarrow a Y - aKY^* + \alpha^2e_t - \alpha^2e_t^e - \alpha^2v_t + au_t + \beta \lambda(\zeta - q_{t-1}) - \beta v_t = 0 \]

\[ \Rightarrow (a^2 + \beta) e_t = a (KY^* - \bar{Y}) + \alpha^2e_t^e + (a^2 + \beta)v_t - \alpha u_t + \beta \lambda(\zeta - q_{t-1}) \]

\[ \Rightarrow e_t = \frac{a}{\alpha + \beta}(KY^* - \bar{Y}) + \frac{\alpha}{\alpha + \beta}e_t^e + v_t - \frac{\alpha}{\alpha + \beta}u_t + \frac{\beta}{\alpha + \beta}(\zeta - q_{t-1}) \]  

This implies that, in a flexible exchange rate regime, the change in exchange rate totally absorbs real exchange rate shock and partially absorbs output shock. Substituting equation (a7) into (a6) yields,

\[ L^{\text{Flex}} = \{ \bar{Y} + \frac{\alpha^2}{\alpha + \beta}(KY^* - \bar{Y}) + \frac{\alpha^3}{\alpha + \beta}e_t^e + \frac{\alpha}{\alpha + \beta}u_t - \frac{\alpha^2}{\alpha + \beta}v_t \]

\[ + \frac{\alpha \beta}{\alpha + \beta}\lambda(\zeta - q_{t-1}) - \alpha e_t^e - \alpha v_t + u_t - KY^*\}^2 + \beta\{ \frac{\alpha}{\alpha + \beta}(KY^* - \bar{Y}) \]

\[ + \frac{\alpha^2}{\alpha + \beta}e_t^e + v_t - \frac{\alpha}{\alpha + \beta}u_t + \frac{\beta}{\alpha + \beta}(\zeta - q_{t-1}) - \lambda(\zeta - q_{t-1}) - v_t\}^2 \]

Canceling similar terms with opposite signs and collecting terms yields,

\[ L^{\text{Flex}} = \{ \frac{\beta}{\alpha + \beta}(KY^* - \bar{Y}) + \frac{\alpha^2}{\alpha + \beta}e_t^e + \frac{\beta}{\alpha + \beta}u_t + \frac{\alpha \beta}{\alpha + \beta}\lambda(\zeta - q_{t-1})\}^2 \]

\[ + \beta\{ \frac{\alpha}{\alpha + \beta}(KY^* - \bar{Y}) + \frac{\alpha^2}{\alpha + \beta}e_t^e - \frac{\alpha}{\alpha + \beta}u_t - \frac{\alpha^2}{\alpha + \beta}\lambda(\zeta - q_{t-1})\}^2 \]

\[ = (\frac{\beta}{\alpha + \beta})^2(\bar{Y} - KY^* + u_t + \alpha \lambda(\zeta - q_{t-1}))^2 \]
\begin{align*}
  & + \beta \left( \frac{-\alpha}{\alpha^2 + \beta} \right)^2 (KY^* - \bar{Y} + \alpha\varepsilon_i - u_i - \alpha\lambda(\zeta - q_{t-1}))^2 \\
  & = \left( \frac{-\beta}{\alpha^2 + \beta} \right)^2 (\bar{Y} - KY^* - \alpha\varepsilon_i + u_i + \alpha\lambda(\zeta - q_{t-1}))^2 \\
  & + \beta \left( \frac{-\alpha}{\alpha^2 + \beta} \right)^2 (\bar{Y} - KY^* - \alpha\varepsilon_i + u_i + \alpha\lambda(\zeta - q_{t-1}))^2 \\
  & = \frac{\beta^2 + \beta\alpha}{(\alpha^2 + \beta)^2} (\bar{Y} - KY^* - \alpha\varepsilon_i + u_i + \alpha\lambda(\zeta - q_{t-1}))^2 \\
  & = \frac{\beta}{\alpha^2 + \beta} (\bar{Y} - KY^* - \alpha\varepsilon_i + u_i + \alpha\lambda(\zeta - q_{t-1}))^2 \\
  & = \frac{\beta}{\alpha^2 + \beta} (\bar{Y} - KY^* - \alpha\varepsilon_i + u_i + \alpha\lambda(\zeta - q_{t-1}))^2 \quad \text{(a8)}
\end{align*}

If exchange rate is fixed it implies that \( \epsilon_i = 0 \) and \( c(c) = 0 \). Substituting these relationships into equation (a8) yields,

\[
L^\text{Fix} = \left\{ \bar{Y} - \alpha\varepsilon_i - \alpha v + u_i - KY^* \right\}^2 + \beta \left\{ -\lambda(\zeta - q_{t-1}) - v_t \right\}^2 \\
= \left\{ \bar{Y} - KY^* - \alpha\varepsilon_i + u_i \right\}^2 + \beta \left\{ \lambda(\zeta + q_{t-1}) + v_t \right\}^2
\]

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